# ON THE STABILITY OF THE NATURAL CIRCULATION SOLAR HEATER

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## ABSTRACT

A theoretical discussion is presented of the stability characteristics of the free convection loop representing the thermosyphonic solar water heater. The system consists of a flat-plate collector, a storage tank and connecting pipes. A one-dimensional model is employed with only one space-coordinate running along the circulation loop. It is assumed that the temperature distributions in the various components of the system are linear and that the flow rate is uniform. Stability of a steady state motion in the system is analyzed by using the linearized stability equations and computing the resulting eigenvalues.

It is found that the regular solar heater is stable when there is no energy utilization from the storage tank. There are perturbation modes which decay very slowly and oscillatory modes with period of the order of the time required to circulate the loop. For strong energy utilization the system becomes unstable.

## NOMENCLATURE

$A$ = cross section

$\hat{B}$ = collector width, m

$c_p$ = specific heat of the fluid, KJ/(kgK)

$\hat{D}$ = diameter, m

$G$ = dimensionless parameter, equation (1)

$g_n$ = acceleration of gravity, m/s$^2$

$\hat{K}$ = overall flow resistance, kg/(m$^5$s)

$L$ = length

$\hat{L}_s$ = overall length of the entire circulation, m

$m = \sum_i \frac{L_i}{A_i}$    $i = c, r, t, d$

$N$ = number of parallel tubes in the collector

$P_0$ to $P_4$ = coefficients of the polynomial equation (11) for $\sigma$

$Q$ = flow rate

$q$ = heat flux per unit length

$\hat{q}_o$ = solar radiation per unit length absorbed in the collector plate, kw/m

$\sigma$ = exponent of the perturbation, equations (8)

$s$ = coordinate along the circulation loop

$T$ = temperature

$\hat{T}_{max} = \hat{q}_o/\hat{U}_c$ = maximum possible system temperature

$\Delta T$ = temperature difference along the collector

$t$ = time

$U = \frac{\hat{U}\hat{L}_s}{\rho c_p \hat{A}_r \hat{v}}$ = overall heat transfer coefficient per unit length

$\hat{v} = (g_n \beta_{th} \hat{T}_{max} \hat{L}_s)^{1/2}$ = characteristic velocity, m/s

$Z$ = height of the system

$z$ = vertical coordinate

$\beta_{th}$ = thermal expansion coefficient, K$^{-1}$

$\gamma$ = utility factor

$\theta$ = temperature perturbation

$\mu$ = dynamic viscosity, kg/(ms)

$\rho$ = density of the fluid, kg/m$^3$

$\phi$ = tilt of the collector relative to the horizon

$\psi$ = flow rate perturbation

### Subscripts

c, d, r, t = collector, downcomer, riser, tank (respectively)

,e = equivalent length representing head losses in piping network

Superscripts

in, out, m = inlet, outlet and average for any system component

Special notations

$\hat{( )}$ = dimensional

$\bar{( )}$ = average steady state value

## INTRODUCTION

Natural circulation loops can sometimes become unstable as shown by Keller (1), Welander (2) and Creveling et al. (3). An instability in a natural circulation solar heater (see Figure 1) would decrease the efficiency of the system because the stratification in the tank may be destroyed and oscillations of the flow rate may cause reversed flow. The existing models describing such solar heaters are based on the assumption of linear temperature distributions in the collector and the tank. Ong (4) presented experimental results(for a special case) showing that the assumption provides a good approximation for the noon time, when the system can be considered to be at a quasi-steady state. Zvirin et al. (5) have shown, by comparison with a more accurate model, that this assumption is justified for the practical range of the solar heater parameters.

The present work suggests a method for the stability study of the natural circulation solar heater. The steady state solution of the governing momentum and energy equations is obtained as in ref.(5).

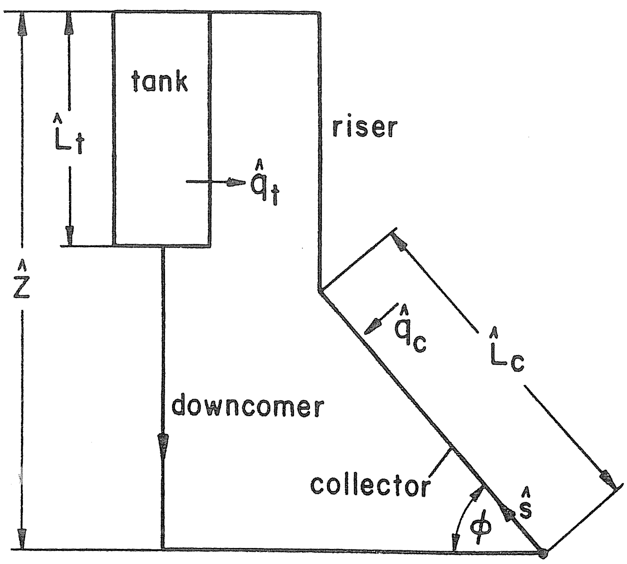


Figure 1 Schematic arrangement of the natural circulation solar heater

The response of the system to a small perturbation is then analyzed by the linear stability theory. It is assumed that the perturbation temperature distribution is also linear. The results demonstrate that, subject to the assumptions made and for the practical values of the system parameters, no instability will occur. However, for large heat losses from the tank, e.g. a heat exchanger instead of a tank, the system can be unstable.

## ANALYSIS

Following Zvirin et al. (5), the system (Figure 1) is represented by a one-dimensional model wherein the coordinate s is taken along the closed circulation loop and T(s) is the mean cross-sectional temperature. It is assumed that the flow rate Q is uniform. The heat transfer coefficients and the properties of the fluid are taken to be constant, except for the buoyancy forces where the density depends on a constant expansion coefficient (Boussinesq approximation).

The variables and parameters are non-dimensionalized, scaling lengths by $\hat{L}_s$, cross sections by $\hat{A}_r$ temperatures by $\hat{T}_{max}$ flow rates by $\hat{A}_r\hat{v}$ and time by $\hat{L}_s/\hat{v}$.

The momentum equation for the entire circulation loop is obtained by piecewise integration over the various parts:

$$m\frac{dQ}{dt} + \frac{1}{G}Q = \oint T dz \qquad (1)$$

where $G = (\rho\beta_{th}g_n\hat{T}_{max})/(\hat{A}_r\hat{v}\hat{K})$ represents the ratio between buoyancy and viscous forces. $\hat{K}$ is the overall flow resistance factor, given by Bartal (6):

$$\hat{K} = \frac{\pi\mu}{8\hat{L}_s}\Big[a_r\Big(\frac{\hat{D}_r\rho\hat{v}}{\mu}Q\Big)^{1-b_r}\Big(\frac{\hat{L}_r + \hat{L}_d + \hat{L}_{r,e}}{\hat{A}_r^{\,2}} + \frac{\hat{L}_t}{\hat{A}_t^{\,2}}\Big) + a_c\Big(\frac{\hat{D}_c\rho\hat{v}\hat{A}_r}{\mu\hat{A}_c}Q\Big)^{1-b_c}\Big(\frac{N\hat{L}_c + \hat{L}_{c,e}}{\hat{A}_c^{\,2}}\Big)\Big] \qquad (2)$$

where the friction coefficient in every part is given by $a/R_e^b$. For laminar flow ($R_e<2000$), a = 64, b = 1, and $\hat{K}$ is constant. For turbulent flow ($R_e>4000$) a = 0.316, b = 0.250, and for the transition regime a = 0.341, b = 0.285.

The energy equation for each part is:

$$A_i \frac{\partial T}{\partial t} + Q \frac{\partial T}{\partial s} = q_i \, [T_i(s)] \qquad i = c, r, t, d \tag{3}$$

where $q_c = U_c\,(1 - T_c)$, $q_t = -U_t T_t$, and it is assumed that $q_r = q_d = 0$. The problem is simplified by the assumption of linear temperature distributions; the partial differential equations (3) then reduce to ordinary ones by integration over each part i (yielding a balance equation):

$$A_i L_i \frac{dT_i^m}{dt} + Q\,(T_i^{out} - T_i^{in}) = L_i q_i \, [T_i^m] \tag{4}$$

where $T_i^m = \frac{1}{2}\,(T_i^{out} + T_i^{in})$.

The temperature is continuous, thus $T^{out}$ of each part is equal to $T^{in}$ of the successive part.

The steady state solution of equations (1) and (4) was given by Zvirin et al. (5) and Bartal (6). The mean temperature in the collector and the tank is:

$$\bar{T}^m = 1/(1 + \gamma) \tag{5}$$

where the utility factor $\gamma$ is defined by:

$$\gamma = U_t L_t/(U_c L_c)$$

The steady state flow rate is:

$$\bar{Q} = U_c L_c \left\{ \frac{\gamma G \, [\frac{Z}{L_c} - \frac{1}{2}\,(\frac{L_t}{L_c} + \sin\phi)]}{(1 + \gamma) U_c} \right\}^{1/2} \tag{6}$$

For laminar flow, G is constant and $\bar{Q}$ can be directly calculated by the last relationship. For turbulent flow in the whole system $b_r = b_c$ and Q can again be directly obtained from equations (6) and (2). There also exists the possibility of different flow regimes in the various parts and equation (6) then must be solved numerically. Finally, the temperature difference along the collector, $\Delta\bar{T}$, is given by:

$$\Delta\bar{T} = \frac{\gamma}{1 + \gamma} \frac{U_c L_c}{Q} \tag{7}$$

Results for the steady state are given in refs. (5) and (6).

For the stability analysis, the flow rate and the temperature distribution are expressed as small perturbations superimposed on the steady state solution:

$$Q = \bar{Q} + \psi e^{\sigma t} \tag{8a}$$

$$T(s,t) = \bar{T}(s) + \theta(s) e^{\sigma t} \tag{8b}$$

The problem consists now of finding the characteristic values of $\sigma$. A positive real part of $\sigma$ indicates instability of the system, while the existence of imaginary parts means oscillations in the system.

Introduction of equations (8) into (1) and (3) and linearizing the resulting equations, we obtain:

$$(m\sigma + \frac{1}{G})\psi = \oint \theta dz \tag{9}$$

$$\sigma A_i \theta_i + \bar{Q} \frac{d\theta_i}{ds} + \psi \frac{d\bar{T}_i}{ds} = - U_i \theta_i$$

$$i = c, r, t, d \tag{10}$$

It is noted that $A_r = A_d = 1$ and $U_r = U_d = 0$. Use is now made of the assumption of linear temperature profiles, both for the steady state $\bar{T}_i(s)$ and for the perturbations $\theta_i(s)$. Integration of equations (10) along each part, using the condition of continuous temperature distribution, gives $\theta_i(s)$ as linear functions of $\psi$, depending also on the steady state solutions $\bar{Q}$ and $\Delta\bar{T}$. The integral in equation (9) can now be performed; it can be seen that $\psi$ is eliminated in this equation, leading to the following characteristic equation for $\sigma$ (the details of the derivation are given by Bartal (6) see appendix B):

$$P_4\sigma^4 + P_3\sigma^3 + P_2\sigma^2 + P_1\sigma + P_o = 0 \tag{11}$$

where the coefficients P of the polynomial are:

$$P_4 = \frac{m\bar{Q}}{4\Delta\bar{T}} \, [A_c L_c L_d \,(A_t L_t + L_r) + A_t L_t L_r \,(A_c L_c + L_d)] \tag{12a}$$

$$P_3 = \frac{m\bar{Q}}{4\Delta\bar{T}}\Big\{[L_tL_c\,(U_tA_cL_d + U_cA_tL_r) + U_cL_cL_d\,(A_tL_t + L_r) + U_tL_tL_r\,(A_cL_c + L_d)]\Big\} - \frac{1}{8}\,(A_cL_c - A_tL_t)\,[Z^2\,(L_r - L_d) + 2Z\times(L_dL_c\sin\phi - L_tL_r) + L_dL_r\,(L_t - L_c\sin\phi) + L_t^2L_r - L_d\,(L_c\sin\phi)^2] + \frac{\bar{Q}}{4G\Delta\bar{T}}\,[A_cL_cL_d(A_tL_t + L_r) + A_tL_tL_r\,(A_cL_c + L_d)] \qquad (12b)$$

$$P_2 = \frac{m\bar{Q}}{4\Delta\bar{T}}\Big\{[4\bar{Q}^2\,(A_tL_t + A_cL_c + L_r + L_d) + U_cL_cU_tL_t\,(L_r + L_d)]\Big\} + \frac{\bar{Q}}{4G\Delta\bar{T}}\,[L_tL_c\times(U_tA_cL_d + U_cA_tL_r) + U_cL_cL_d(A_tL_t + L_r) + U_tL_tL_r\,(A_cL_c + L_d)] - \frac{\bar{Q}}{8}\Big\{4\,[Z^2(L_r + L_d) - 2Z\times(L_rL_t + L_dL_c\sin\phi + L_rL_d) + L_rL_t^2 + L_dL_c^2\sin^2\phi] + (L_c\sin\phi + L_t - 2Z)\,(A_cL_c - A_tL_t)\,(L_r - L_d) + (A_cL_c + A_tL_t)\,[4Z^2 - 2Z\,(2L_t + 2L_c\sin\phi + L_d + L_r) + (L_t - L_c\sin\phi)(L_d - L_r) + 2(L_t^2 + L_c^2\sin^2\phi)]\Big\} - \frac{1}{8}\,(U_cL_c - U_tL_t)\,[Z^2\,(L_r - L_d) + 2Z\,(L_dL_r\sin\phi - L_tL_r) + L_rL_t^2 - L_dL_c^2\sin^2\phi + L_rL_d(L_t - L_c\sin\phi)] \qquad (12c)$$

$$P_1 = \frac{m\bar{Q}^3}{\Delta\bar{T}}\,(U_cL_c + U_tL_t) + \frac{\bar{Q}}{4G\Delta\bar{T}}\,[4\bar{Q}^2\,(A_tL_t + A_cL_c + L_r + L_d) + U_cL_cU_tL_t\,(L_r + L_d)] - \frac{\bar{Q}}{8}\Big\{(L_c\sin\phi + L_t - 2Z)\,[4\bar{Q}\,(A_cL_c + A_tL_t + L_r + L_d) + (U_cL_c - U_tL_t)\,(L_r - L_d)] + (U_cL_c + U_tL_t)\,[4Z^2 - 2Z\,(2L_t + 2L_c\sin\phi - L_d - L_r) + 2\,(L_t^2 + L_c^2\sin^2\phi) + (L_t - L_c\sin\phi)\times(L_d - L_r)]\Big\} \qquad (12d)$$

$$P_0 = \frac{\bar{Q}^2}{2}\,(U_cL_c + U_tL_t)\,\Big[\frac{2\bar{Q}}{G\Delta\bar{T}} - L_c\sin\phi - L_t + 2Z\Big] \qquad (12e)$$

For any given set of coefficients P, computation of the roots $\sigma$ of equation (11) is quite simple. The influence of the numerous parameters on the stability of the system (on the roots $\sigma$) was investigated by Bartal (6). His results indicate that the dominant parameter is the utility factor $\gamma$. The roots $\sigma$ of equation (11) for various values of $\gamma$ in a typical solar water heater (the data is given in appendix A) are given in table 1. The table also includes the steady state flow rate $\bar{Q}$. It can be seen that the system is marginally stable at $\gamma = 780$, and the instability is of the oscillatory mode. For this case, the flow is turbulent in the riser and downcomer ($Re \sim 5600$) and laminar in the collector ($Re \sim 600$). When $\gamma$ is small there is one mode of perturbation which decays slowly; when $\gamma = 0.1$, for example, the larger decay time is $1.6 \times 10^4$s. The period of the oscillations is found to be of the same order as the time it takes to circulate the entire loop. This result was also obtained by Creveling et al. (3). It is interesting to note that even for very large values of $\gamma$, the system is still unstable. Note also that there is a range near $\gamma = 1000$ where all the roots are complex, whereas in most of the range of $\gamma$ two roots are real. *

As was shown by Bartal (6), instabilities in the system appear only at very high values of the utility factor $\gamma$ for the practical range of all the other system parameters. This means that the regular system used to accumulate hot water in the tank is stable, but when a heat exchanger is employed instead of the tank, instabilities can occur.

Table 1. Roots $\sigma$ of equation (11) for various values of $\gamma$, the other parameters are given in appendix A.

| $\gamma$ | $\bar{Q}$ | $\sigma_1$ | $\sigma_2$ | $\sigma_3, \sigma_4$ |
|---|---|---|---|---|
| 0.01 | 0.013 | -0.218 | -0.00022 | -0.010 ± 0.035i |
| 0.1 | 0.034 | -0.289 | -0.00024 | -0.016 ∓ 0.096i |
| 1 | 0.060 | -0.514 | -0.00044 | -0.024 ∓ 0.169i |
| 10 | 0.075 | -0.601 | -0.0024 | -0.028 ∓ 0.210i |
| 100 | 0.077 | -0.608 | -0.024 | -0.030 ∓ 0.209i |
| 500 | 0.077 | -0.572 | -0.183 | -0.016 ∓ 0.177i |
| 700 | 0.077 | -0.537 | -0.287 | -0.0034 ∓ 0.173i |
| 780 | 0.077 | -0.513 | -0.330 | 0.0 ∓ 0.173i |
| 800 | 0.077 | -0.506 | -0.349 | 0.0011 ∓ 0.173i |
| 1000* | 0.077 | -0.457 | ±0.103i | 0.0079 ∓ 0.173i |
| 10000 | 0.077 | -2.19 | -0.823 | 0.0390 ∓ 0.187i |
| 100000 | 0.077 | -22.6 | -0.762 | 0.0430 ∓ 0.191i |

* For $\gamma = 1000$ $\sigma_1, \sigma_2$ are conjugate complex pair: -0.457 ± 0.103i.

## REFERENCES


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## APPENDIX A

### Data For a Typical Solar Water Heater (Figure 1)

Geometrical parameters:

$\hat{L}_c = 1.8m$, $\hat{L}_t = 0.7m$, $\hat{L}_s = 7.0m$, $\hat{Z} = 2.2m$, $\phi = 45°$,

$\hat{B} = 1.4m$, $\hat{D}_r = 0.0190m$, $\hat{D}_c = 0.0127m$,

$N = 14$, $\hat{D}_t = 0.50m$.

Water properties:

$c_p = 4.2KJ/(kgK)$, $\mu = 4.5x10^{-4}kg/(ms)$,

$\rho = 10^3 kg/m^3$, $\beta_{th} = 6.3x10^{-4}k^{-1}$.

Average Solar Radiation, Noon Time at 32°N - $0.9kw/m^2$, $\hat{q}_o = 0.9x\hat{B} = 1.25kw/m$.

Overall Heat Transfer Coefficients:

$\hat{U}_c = 0.018kw/(mk)$, $\hat{U}_{t\ losses} = 1.4x10^{-3}kw/(mk)$.

## APPENDIX B

### The Derivation of Equation (11)

### According to Bartal (6)

Integration of equations (10) and rearranging gives, along the collector:

$$\overline{Q}\,\Delta\theta_c + L_c\theta_c^m(\sigma A_c + U_c) = -\psi\Delta\overline{T} \qquad (B.1)$$

along the storage tank:

$$\overline{Q}\,\Delta\theta_c + L_t\theta_t^m(\sigma A_t + U_t) = \psi\Delta\overline{T} \qquad (B.2)$$

and along the connecting pipes:

$$\overline{Q}\,\Delta\theta_i + \sigma L_i\theta_i^m = 0 \quad (i = r,d) \qquad (B.3)$$

where:

$$\Delta\theta_i = \theta_i^{out} - \theta_i^{in} \qquad (i = c,t,r,d) \qquad (B.4)$$

$$\theta_i^m = \frac{1}{2}(\theta_i^{out} + \theta_i^{in}) \qquad (B.5)$$

Continuity of temperatures yields to the solution of $\theta_i(s)$ from equations (B.1)-(B.3), as follows:

$$\theta_c^{in} = \theta_d^{out} = y_1\theta_t^{out} \qquad (B.6)$$

$$\theta_c^{out} = \theta_r^{in} = \psi\Delta\overline{T}\,\frac{y_1y_2 - y_3}{y_7} \qquad (B.7)$$

$$\theta_t^{in} = \theta_r^{out} = y_4\theta_c^{out} \qquad (B.8)$$

$$\theta_t^{out} = \theta_d^{in} = \psi\Delta\overline{T}\,\frac{y_5 - y_4y_6}{y_7} \qquad (B.9)$$

where:

$$y_1 = \frac{\overline{Q} - \sigma L_d/2}{\overline{Q} + \sigma L_d/2} \qquad (B.10)$$

$$y_2 = \overline{Q} - \sigma A_cL_c/2 - U_cL_c/2 \qquad (B.11)$$

$$y_3 = \overline{Q} + \sigma A_tL_t/2 + U_tL_t/2 \qquad (B.12)$$

$$y_4 = \frac{\overline{Q} - \sigma L_r/2}{\overline{Q} + \sigma L_r/2} \qquad (B.13)$$

$$y_5 = \overline{Q} + \sigma A_cL_c/2 + U_cL_c/2 \qquad (B.14)$$

$$y_6 = \overline{Q} - \sigma A_tL_t/2 - U_tL_t/2 \qquad (B.15)$$

$$y_7 = y_3y_5 - y_1y_2y_4y_6 \qquad (B.16)$$

Substitution of equations (B.6)-(B.9) into equation (9), and integrating along the circulation loop, gives the polynomial in equation (11).